

Reply on “Fluctuation-dissipation considerations for phenomenological damping models for ferromagnetic thin films” [N. Smith, J. Appl. Phys. 92, 3877 (2002)]

Vladimir L. Safonov and H. Neal Bertram
 Center for Magnetic Recording Research,
 University of California - San Diego,
 9500 Gilman Drive, La Jolla, CA 92093-0401
 (February 1, 2008)

We show that the critique of our recent papers presented in the abovementioned paper (NS) appeals to an incorrect mathematical analogy between electrical circuits and linear magnetization dynamics, improperly uses classical concepts of normal modes and basic equations, gives inconsistent results and therefore comes to incorrect conclusions.

The study of linear stochastic magnetization dynamics is of great importance in applications to nano-magnetic devices based on ultra-thin films. Gyromagnetic magnetization motion around an effective field is randomly forced by fluctuations on spins by means of interaction with a thermal bath (phonons, magnons, conduction electrons, impurities, etc.). The role of the thermal bath can be approximately reduced to terms describing relaxation and random fields in the magnetization dynamic equations.

For simplicity, the conventional theoretical approach is purely phenomenological. It is based on the Landau-Lifshitz equation [1] with random fields, or its modification in a Gilbert form [2]. This approach has the defect that the phenomenological damping term in Landau-Lifshitz (and Gilbert) equation was introduced just for the case of high magnetic symmetry (axial symmetry). An applicability of an isotropic damping together with corresponding noise to strongly anisotropic system such as thin film has not been proven [3].

In our recent papers [4], [5] we have developed a theoretical approach based upon the representation of the magnetization dynamics as the motion of damped non-linear oscillator driven by a random force (thermal fluctuations). The oscillator model is a convenient tool [6] to establish a “bridge” between the microscopic physics, where the oscillator variables a^* and a naturally describe spin excitations (as creation and annihilation operators), and the macroscopic stochastic magnetization dynamics for normal modes. We have calculated the magnetization noise spectrum in a thin film for a physical loss mechanisms [7] and shown that our result differs from that of obtained by the phenomenological approach and is in agreement with experiment [8].

Recently Neil Smith (the paper cited in the title, later, NS) criticized our approach claiming that “a proper understanding of the FDT (fluctuation-dissipation theorem) ... may have been complicated by several recent papers [4], [5]”. In this Reply we argue that this critique 1) appeals to an incorrect mathematical analogy between

electrical circuits and linear magnetization dynamics, 2) improperly uses classical concepts of normal modes and basic equations claiming simultaneous diagonalization of three hermitian matrices, and 3) gives inconsistent results.

1. *Incorrect mathematical analogy.* Any analogy in physics is based on a similar structure of mathematical equations. The basic equations for linear electric circuits have the form (here and later we use notations of NS):

$$\sum_{j=1}^N \left[M_{ij} \frac{d^2 X_j}{dt^2} + D_{ij} \frac{dX_j}{dt} + K_{ij} X_j \right] = F_i(t). \quad (1)$$

This is a set of the second order differential equations, where the scalar variables X_j describe electric charges, $F_i(t)$ are external voltages and symmetric matrices describe inductances (M_{ij}), resistors (D_{ij}) and capacitors (K_{ij}).

On the other hand, linear magnetization dynamics do not contain inertial terms ($M_{ij} = 0$) and do contain an antisymmetric gyromagnetic matrix G_{ij} :

$$\sum_{j=1}^N \left[(D_{ij} - G_{ij}) \frac{dX_j}{dt} + K_{ij} X_j \right] = F_i(t). \quad (2)$$

This is a set of first order differential equations and there is no mathematical analog for G_{ij} in a linear electric scheme. In order to describe N coupled magnetic oscillators (spin waves) each dynamic variable X_j should be chosen as a column (not a scalar), containing two transverse magnetization components in the j -th micromagnetic cell:

$$X_j = \begin{pmatrix} m_{x,j} \\ m_{y,j} \end{pmatrix}. \quad (3)$$

In this formulation there is no direct mathematical analogy between linear electrical circuits (1) and linear magnetization dynamics (2). Thus, without loss of generality, the compilation of “electrical analogies” (Sec.III in NS) has no mathematical consequences for magnetic systems [9].

2. *Incorrect claim of simultaneous diagonalization of three hermitian matrices (including damping $\overleftrightarrow{\mathbf{D}}$).* We

think that this claim is just a simple misunderstanding in NS of the normal mode (spin wave) concept in magnetic dynamics. Spin waves are a principal concept of a conservative magnetic system where the energy of the linear magnetic oscillations can be diagonalized and represented as a set of independent harmonic oscillators (see, e.g., [6]). Uniform rotation of the magnetization corresponds to a spin wave with zero momentum. Spin waves interact with each other and with other degrees of freedom (elastic waves, conduction electrons, etc.). Spin wave damping appears as a result of reducing (averaging) these interactions (interaction with a thermal bath). Therefore the reduced equations can not have a general structure. The damping matrix is specific to the magnetic system and can not have an arbitrary form, as assumed in NS.

3. *Inconsistency of obtained “general” results.* Let us check the general results obtained in NS for the fluctuation-dissipation theorem as used in Gilbert and Bloch-Bloembergen dynamics. In NS the Gilbert equation is written in the form:

$$(\overleftarrow{\mathbf{D}} - \overleftarrow{\mathbf{G}}) \cdot \frac{d\overrightarrow{\mathbf{m}}}{dt} + \overleftarrow{\mathbf{H}} \cdot \overrightarrow{\mathbf{m}} = \overrightarrow{\mathbf{h}}(t), \quad (4)$$

where

$$\overleftarrow{\mathbf{D}} = \frac{\alpha}{\gamma} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \overleftarrow{\mathbf{G}} = \frac{1}{\gamma} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \quad (5)$$

and $\overleftarrow{\mathbf{H}}$ describes effective magnetic fields. The corresponding thermal fluctuation fields are given by:

$$\langle \overrightarrow{\mathbf{h}}(t_0 + \tau) \overrightarrow{\mathbf{h}}(t_0) \rangle_{ij} = \frac{2kT\alpha}{\gamma M_s \Delta V} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \delta_{ij} \delta(\tau), \quad (6)$$

referred as FDT in NS. It is well known that the Gilbert equation can be rewritten in a mathematically equivalent form of the Landau-Lifshitz equation. This procedure is most simple in the case of small damping when we can neglect the higher order damping terms (such as α^2). We can find the magnetization derivative on time from Eq.(4) without damping

$$\frac{d\overrightarrow{\mathbf{m}}}{dt} = (-\overleftarrow{\mathbf{G}})^{-1} \cdot (-\overleftarrow{\mathbf{H}} \cdot \overrightarrow{\mathbf{m}} + \overrightarrow{\mathbf{h}}(t)) \quad (7)$$

and represent the damping term as

$$\begin{aligned} \overleftarrow{\mathbf{D}} \cdot \frac{d\overrightarrow{\mathbf{m}}}{dt} &= \overleftarrow{\mathbf{D}} \cdot (-\overleftarrow{\mathbf{G}})^{-1} \cdot (-\overleftarrow{\mathbf{H}} \cdot \overrightarrow{\mathbf{m}} + \overrightarrow{\mathbf{h}}(t)) \\ &= \alpha\gamma \left(-\overleftarrow{\mathbf{G}} \cdot \overleftarrow{\mathbf{H}} \cdot \overrightarrow{\mathbf{m}} + \overleftarrow{\mathbf{G}} \cdot \overrightarrow{\mathbf{h}}(t) \right). \end{aligned} \quad (8)$$

Here we utilize (5) and take into account that $(-\overleftarrow{\mathbf{G}})^{-1} = \gamma^2 \overleftarrow{\mathbf{G}}$. Neglecting small $\alpha\gamma \overleftarrow{\mathbf{G}} \cdot \overrightarrow{\mathbf{h}}(t)$, the Eq.(4) may be written as:

$$-\overleftarrow{\mathbf{G}} \cdot \frac{d\overrightarrow{\mathbf{m}}}{dt} + (\overleftarrow{\mathbf{H}} - \overleftarrow{\mathbf{G}} \cdot \alpha\gamma \overleftarrow{\mathbf{H}}) \cdot \overrightarrow{\mathbf{m}} = \overrightarrow{\mathbf{h}}(t). \quad (9)$$

On the other hand, in NS the Bloch-Bloembergen equation has been written as:

$$-\overleftarrow{\mathbf{G}} \cdot \frac{d\overrightarrow{\mathbf{m}}}{dt} + (\overleftarrow{\mathbf{H}} - \overleftarrow{\mathbf{G}}/T_2) \cdot \overrightarrow{\mathbf{m}} = \overrightarrow{\mathbf{h}}(t) \quad (10)$$

with corresponding FDT of the form:

$$\langle \overrightarrow{\mathbf{h}}(t_0 + \tau) \overrightarrow{\mathbf{h}}(t_0) \rangle = \frac{kT}{\gamma M_s \Delta V} \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \delta_{ij} \frac{\text{sgn}(\tau)}{T_2}. \quad (11)$$

We see that Eqs.(9) and (10) have similar forms. They coincide, for example, in the case

$$\overleftarrow{\mathbf{H}} = H_0 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad (12)$$

and $1/T_2 = \alpha\gamma H_0$. In this case their FDT relations (6) and (11) are distinctly different from each other. This fact simply indicates that NS results are inconsistent.

This work was partly supported by matching funds from the Center for Magnetic Recording Research at the University of California - San Diego and CMRR incorporated sponsor accounts.

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[9] In principle, it would be interesting to construct a correct mathematical analogy between linear electrical circuits and linear magnetization dynamics using appropriate choice of variables in magnetic system. This, however, has not been done in NS.